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A reduced self-calibrating method for rotary table error motions

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Abstract

A novel method for the determination of rotary table errors with six degrees of freedom on coordinate measuring machines and machine tools is introduced. The method is based on the measurement of an uncalibrated circular ball plate in different rotational positions on the rotary table, which is also referred to as the three-rosette method. The novel procedure allows the number of measurement positions to be reduced significantly compared to the complete three-rosette method, while the measurement uncertainty still remains sufficiently small. The method relies on error separation and is self-calibrating, which means that no external reference is required. The sophisticated design of a new ball plate allows rotary table errors to be determined in angular steps of only 5°.

Keywords: rotary tables, error mapping, coordinate metrology, coordinate measuring machine

(Some figures may appear in colour only in the online journal)

1. Introduction

Coordinate measuring machines (CMMs) are often equipped with rotary tables which expand the machine's kinematics to include a fourth axis. This is beneficial for the measurement of rotationally symmetric workpieces, as multi-stylus configurations can be avoided. However, the additional axis introduces further sources of errors that must be investigated before a numerical correction or proper uncertainty analysis can be carried out. A broad variety of methods are used for the measurement of rotary table deviations. The position error *crz*, i.e. the rotational error around the axis of rotation, is often measured using an autocollimator, while other errors can be detected by measuring the movement of spheres mounted in the centre of the rotary table via displacement sensors [1–3]. In [4], a method is proposed which allows the measurement of all six rotary table error components directly on the CMM or machine tool by using a tracking interferometer and sequential multilateration.

For the determination of machine errors, self-calibration strategies based on error separation are favourable, as no externally calibrated measurement standard is needed [5]. In [6], a method is introduced which uses a simple and uncalibrated cylindrical artefact to separate the rotary axis errors from those of the linear axes. However, this method is only capable of identifying two of the six error motion components assigned to rotary axes. The procedure presented in [7] uses the so-called three-rosette method applied to a circular ball plate and provides all six error motion components. The resolution of the errors obtained (i.e. the angular step size), however, depends on the design of the ball plate. The ball plate used in [7] is equipped with 12 equally distributed spheres and therefore yields error motions with a step size of 30°. This may be unsatisfactory if the results are to be used, for example, for compensation of gear measurements, as large parts of the compensation data must be calculated by means of interpolation [8, 9]. Putting more spheres on the circular ball plate will improve the resolution but also increase the measurement time and production costs for the artefact.

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The method presented in this paper builds on the procedure from [7] combined with a reduced measurement effort and a novel circular ball plate with a sophisticated design. The work is inspired by [10], where the authors reported on a reduced three-rosette method with one degree of freedom (DOF) for pitch calibration of gears.

The paper starts with a description of the mathematical background of the three-rosette method with six DOF in section 2. The design of the novel circular ball plate is presented in section 3. Section 4 introduces the measurement setup used for validation and its results are given in section 5. Section 6 concludes the paper.

2. The three-rosette method with six DOF

Every measurement performed using a CMM with a rotary table is influenced by deviations from the CMM's linear axes, by rotary table errors, and by the manufacturing errors of the artefact to be measured. The principle on which the three-rosette method is based is to separate these three error sources from one another by means of measurements of an artefact in multiple orientations. It should be noted that this technique is only capable of detecting systematic errors; hence, stochastic errors (for example, those of the probing system) should ideally be as small as possible. Moreover, the deviations of some rotary tables are not 2π-periodic, as is typically the case for rotary tables with rolling-element bearing. While the three-rosette method can still be used to detect the rotary table deviations using an adapted measurement strategy [8, 9], the results generally cannot be used for compensation purposes, since the angular position of the rotary table is known only modulo 2π. In this paper, it is assumed that the deviations are periodic after one revolution, as is normally the case for highly precise rotary tables with aerostatic bearings.

In [10], a reduced three-rosette method with one DOF was presented to determine the pitch deviations of gears. The same setup could, in principle, also be used to provide rotary table errors. However, with this method, only the positioning error crz can be detected, which is only one of the six rotary table error motions.

The circular ball plate introduced in [7] is a simple artefact that represents 3D positions on a divided circle. It is one of the easiest to realise designs available which allows the complete six DOF rotary table errors to be inspected. The measuring procedure for the complete method is as follows: first, all the balls are measured at the first position in the CMM by moving them one by one into position with the rotary table. Then, the CMM moves one angular step (in this case given by the angular steps of the balls) to the second measuring position, and again all balls are measured. The procedure is continued in this way until every ball has been measured in all CMM positions. Figure 1 shows the measurement principle for a plate with six balls.

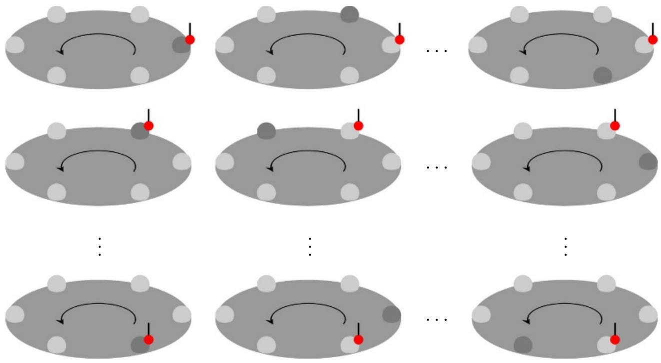

Figure 1. Measurement principle of the complete three-rosette method. In each row, the CMM position remains constant, whereas in each column, the sphere to be measured stays the same.

In the reduced method, either the number of balls or the number of CMM positions to be measured (or both) can be decreased. In figure 1, reducing the number of balls would mean removing some of the rows in the measurement pattern, while reducing the CMM positions would mean removing some of the columns.

The three-rosette method with six DOF uses a model which describes the results of the measurements of a ball plate on a rotary table as a combination of the nominal coordinates and the influence of the deviations of the rotary table, the CMM, and the ball positions on the artefact, including clamping deviations. By means of a least squares fit, these different deviations can be separated from each other, thus allowing the rotary table error motions to be obtained. It must be noted that, since all lengths are measured with the CMM, a scale factor of the linear axis will affect the results. However, the resulting relative errors are very small and their influence on the rotary table deviations can be neglected. Nevertheless, the method does not provide a calibration of the size of the ball plate.

In the following, the model used to describe the influence of the deviations on the measurement is explained. The coordinate system is fixed to the CMM, with its z axis given by the axis of rotation. The x and y axes are given by the corresponding axes of the CMM. The z component of the origin is at the height of the centres of the balls on the plate.

2.1. Rotary table deviations

A rotary table has six DOF: three translational deviations and three rotational deviations. These deviations are measured at N angular positions uniformly distributed over a complete rotation.

If the nominal axis of rotation is parallel to the z axis of the CMM, the translational deviations are denoted by ctx_i, cty_i, ctz_i and the rotational deviations by crx_i, cry_i, crz_i for each $i = 0, 1, \ldots, N-1$. These deviations are combined to form two vectors

$$c_i = \begin{pmatrix} crx_i \\ cry_i \\ crz_i \end{pmatrix} \qquad d_i = \begin{pmatrix} ctx_i \\ cty_i \\ ctz_i \end{pmatrix} \tag{1}$$

for each $i = 0, 1, \ldots, N-1$. A point fixed to the rotary table which has the nominal coordinates p at zero position of the

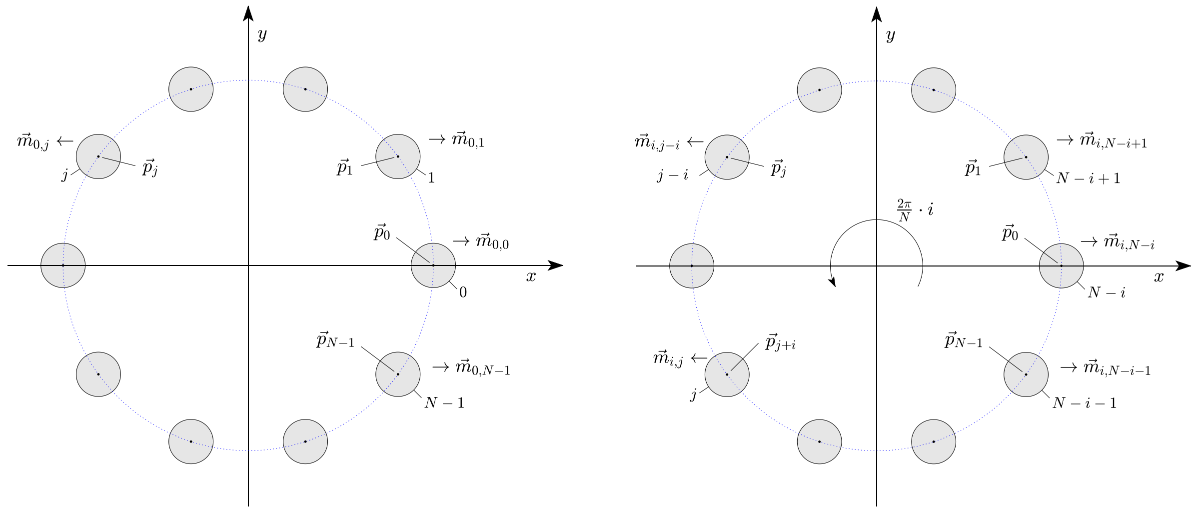

Figure 2. Left: The ball plate in the rotary table angle position $\varphi = 0$. The origin of the coordinate system is on the axis of rotation, the z position given by the ball centre position. The position of ball zero lies on the positive x axis. Right: the ball plate is rotated by i angular steps, i.e. the rotary table is in position $\varphi = \frac{2\pi}{N} \cdot i$. The ball with index j is then rotated to the position p_{i+j} in the CMM,

error-free rotary table has the actual coordinates

$$p' = (\mathrm{I} + \hat{c}_i) \cdot B_i \cdot p + d_i \tag{2}$$

at the angular position $\frac{2\pi}{N} i$. Here, for $x \in \mathbb{R}^3$, $\hat{\cdot}$ is defined by

$$\hat{x} = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}. \tag{3}$$

B_i is the rotation around the z-axis given by

$$B_i = \begin{pmatrix} \cos(\frac{2\pi}{N} i) & -\sin(\frac{2\pi}{N} i) & 0 \\ \sin(\frac{2\pi}{N} i) & \cos(\frac{2\pi}{N} i) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

and I denotes the 3×3 identity matrix.

2.2. Ball plate deviations

At the zero position of the ideal rotary table, the spheres on the ball plate have the nominal centre point positions $p_j = B_j \cdot p_0$, where $p_0 = (\rho, 0, 0)$ is the position of the first ball and ρ is the radius of the circle on which the sphere centres are located. The deviations from the nominal positions p_j are denoted by

$$a_j = \begin{pmatrix} a_{j,x} \\ a_{j,y} \\ a_{j,z} \end{pmatrix}. \tag{5}$$

These deviations are a superposition of the manufacturing errors of the balls on the plate and of clamping errors of the plate on the rotary table. The actual positions of the ball centre point of ball j at the rotary table angular position i of the balls are thus given by

$$(\mathrm{I} + \hat{c}_i) \cdot B_i \cdot (p_j + a_j) + d_i. \tag{6}$$

2.3. CMM deviations

Further let the systematic deviations of the CMM at the positions p_k be given by

$$b_k = \begin{pmatrix} b_{k,x} \\ b_{k,y} \\ b_{k,z} \end{pmatrix} \tag{7}$$

for $k = 0, 1, \ldots, N-1$. Then, the actual position of the centre point of ball j at the rotary table angular position i is obtained by

$$(\mathrm{I} + \hat{c}_i) \cdot B_i \cdot (p_j + a_j) + d_i + b_{i+j}. \tag{8}$$

2.4. Deviation of the coordinate system

Since the position and orientation of the rotary axis are not known exactly, an additional transformation of the coordinate system with rotation matrix U and transformation $v \in \mathbb{R}^3$ must be taken into account. The actual position of ball j of the ball plate at the angular position i of the rotary table is then given by

$$U \cdot [(\mathrm{I} + \hat{c}_i) \cdot B_i \cdot (p_j + a_j) + d_i + b_{i+j}] + v. \tag{9}$$

Since the rotation U is close to the identity, it can be approximated with $U \approx \mathrm{I} + \hat{u}$ with $u \in \mathbb{R}^3$. If the quadratic terms are neglected, it follows that the measured position m_{ij} of ball j of the ball plate at the angular position i of the rotary table is given by

$$\begin{aligned} m_{ij} = & p_{i+j} + \hat{c}_i \cdot p_{i+j} + B_i \cdot a_j \\ & + d_i + b_{i+j} + \hat{u} \cdot p_{i+j} + v. \end{aligned} \tag{10}$$

Figure 2 shows the ball plate and the resulting measurements in start position and in rotated position together with the resulting ball centre measurements m_{ij}.

For the complete three-rosette method, where all N^2 measurements are available, the model parameters a_j, b_k, c_i, d_i, u and v for $i,j,k = 0,1,\ldots,N-1$ are estimated via a least squares fit, that is, by minimizing the sum

$$\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \|p_{i+j} + \hat{c}_i \cdot p_{i+j} + B_i \cdot a_j + d_i + b_{i+j} + \hat{u} \cdot p_{i+j} + v - m_{ij}\|^2. \tag{11}$$

For the reduced three-rosette method, measurements are taken only for a selection $\mathcal{R} \subseteq \{0,1,\ldots,N-1\}$ of $R \leqslant N$ balls on the plate, or only for a selection $\mathcal{S} \subseteq \{0,1,\ldots,N-1\}$ of $S \leqslant N$ CMM positions (or both). To this end, the index $k = i+j$, which denotes the CMM position, is used instead of i. This yields

$$m_{k-j,j} = p_k + \hat{c}_{k-j}p_k + B_{k-j}a_j + d_{k-j} + b_k + \hat{u}p_k + v. \tag{12}$$

The parameters are then obtained by minimizing the sum

$$\sum_{j\in\mathcal{R}}\sum_{k\in\mathcal{S}} \|p_k + \hat{c}_{k-j}p_k + B_{k-j}a_j + d_{k-j} + b_k + \hat{u}p_k + v - m_{k-j,j}\|^2. \tag{13}$$

Such a linear least-squares problem can be written as a matrix equation

$$\|T\cdot\xi - \zeta\|^2 \rightarrow \min, \tag{14}$$

where $\xi \in \mathbb{R}^{3R+3S+6N+6}$ is the parameter vector consisting of the variables a,b,c,d,u and v, T is the $3RS \times (3R+3S+6N+6)$ matrix describing the model, and $\zeta \in \mathbb{R}^{3RS}$ a vector containing the differences between the measurements and the nominal positions $m_{k-j,j} - p_k$.

However, in order to obtain a unique solution comparable to results from other methods, some additional constraints must be applied.

2.5. Constraints

- To set the coordinate system, the actual CMM positions $p_k + b_k$ are optimally fitted to the nominal positions p_k, for $k \in \mathcal{S}$:

$$\sum_{k\in\mathcal{S}} b_k = 0, \qquad \sum_{k\in\mathcal{S}} p_k \times b_k = 0. \tag{15}$$

- To prevent a scaling of the coordinate system, the following equation must be satisfied:

$$\sum_{k\in\mathcal{S}} p_k^t b_k = 0. \tag{16}$$

- The axis of rotation is chosen in such a way that the mean values of the deviations are vanishing:

$$\sum_{i=0}^{N-1} c_i = 0, \qquad \sum_{i=0}^{N-1} d_i = 0. \tag{17}$$

- To eliminate the effect of an inclination and shift of the ball plate on the rotary table deviations, the following constraints are applied:

$$\sum_{i=0}^{N-1} B_i^t c_i = 0, \qquad \sum_{i=0}^{N-1} B_i^t d_i = 0. \tag{18}$$

  Here, only the equations for the x and y components are relevant; the equations for the z components of the rotary table deviations are already included in the previous constraints.
- The deviations of the ball plate are calculated in such a way that rotational deviations around the z axis and position deviations in z direction are on average zero:

$$\sum_{j\in\mathcal{R}} \begin{pmatrix} -\sin\left(\frac{2\pi}{N}j\right) & \cos\left(\frac{2\pi}{N}j\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} a_j = 0. \tag{19}$$

For appropriate choices of $\mathcal{R}$ and $\mathcal{S}$, equation (14) together with the constraints gives unique solutions for all parameters. Reference is made to [10, 11] for some necessary conditions for choices of the ball and CMM positions. However, in this setting, appropriate choices may be defined by the requirement that the matrix formed by the constrained linear system given by equation (14) together with the above constraints must have full rank. If the constraints are written as a matrix equation $G\cdot\xi = 0$ with a $19\times(3R+3S+6N+6)$ matrix G, then the solution can be calculated by

$$\begin{pmatrix} T^tT & G^t \\ G & 0 \end{pmatrix}\begin{pmatrix} \xi \\ \lambda \end{pmatrix} = \begin{pmatrix} T^t\zeta \\ 0 \end{pmatrix} \tag{20}$$

with Lagrangian multiplier λ.

Uncertainties for all parameters can be derived from the constrained linear least squares system. This follows several well-known procedures as explained, for example, in [12]. The covariance matrix for the calculated parameters is obtained by $V = \sigma^2 \cdot V_0$, where the matrix V_0 is given by

$$\begin{pmatrix} V_0 & * \\ * & * \end{pmatrix} = \begin{pmatrix} T^tT & G^t \\ G & 0 \end{pmatrix}^{-1}. \tag{21}$$

Here, V_0 denotes the upper left $(3R+3S+6N+6)\times(3R+3S+6N+6)$ block of the matrix, while the other blocks are not of interest and are denoted by $*$. The standard deviation σ of a single ball centre measurement is estimated by $\sigma^2 = \frac{w^tw}{\nu}$, with the residuals $w = \zeta - T\xi$ and the DOF ν. (If the standard deviation of a single ball centre measurement can be estimated by other, independent means such as knowledge of the measurement capability of the CMM, then this value can also be used.)

As in [10, 11], the uncertainties depend not only on the number of measured balls and positions but also on the choice of the positions. There is no (efficient) algorithm known by the authors to determine which choice of positions will yield smallest possible uncertainties. However, the principal aim is simply to avoid choices which lead to especially high uncertainties; this can easily be achieved by simply calculating the covariance matrix V for a preliminary assumed value of σ to test a small set of positions.

It must be noted that the uncertainties given by the covariance matrix are calculated based on the assumption that each ball centre measurement is independently and identically normally distributed, that the assumed error model is completely fulfilled, and that no other sources contribute to the measurement uncertainty. Since this assumption in realistic situations is never completely fulfilled, the measurement uncertainties may be higher than the values obtained by means of the covariance matrix V.

3. Design of a new circular ball plate

For practical application, the situations in which either the number of balls on the plate or the number of measured CMM positions is reduced are of most interest. If the user already has a ball plate with N balls on a regular angular grid, the number of CMM positions could be reduced to some $S < N$ in order to also reduce the required measurements from N^2 ball measurements to $S \cdot N$ measurements. However, since in this case N is the number of balls on the plate, N is typically a rather small number. The ball plate used in [7], for example, has 12 balls and therefore allows the rotary table deviations to be detected in steps of 30°. Achieving a 5° resolution for the rotary table deviations would require a ball plate with 72 balls, which is not only expensive due to the large number of highly precise spheres needed, but may be even impossible to construct in such a way that plate and sphere radii can be combined in a suitable way. It is therefore preferable to reduce the number R of balls measured in such a way that only a subset $\mathcal{R} = \{j_1, j_2, \dots, j_R\}$ of all balls are measured, with $R < N$. If measurements are performed on all CMM positions $\mathcal{S} = \{0, 1, \dots, N-1\}$, a necessary condition for the unique solvability of equation (20) is given by

$$\gcd(j_2 - j_1, j_3 - j_1, \dots, j_R - j_1, N) = 1, \qquad (22)$$

where gcd denotes the greatest common divisor. Of course, this condition can easily be fulfilled, e.g. if $j_1 = 0$ and $j_2 = 1$. However, it is not necessary to have two numbers j_r and j_{r+1} with difference $j_{r+1} - j_r = 1$. The condition is also fulfilled if $j_1 = 0, j_2 = 3$ and $j_3 = 7$, for example. This is important when a suitable ball plate for the reduced method is designed, where it might not be possible to mount two balls with the distance of one step with respect to the underlying angular grid on the plate.

If condition (22) is not fulfilled and the greatest common divisor is some number $\mu \geqslant 2$, then the distance between any two ball positions is a multiple of μ. Equation (13) splits then into μ parts, so that deviations of the rotary table with a period of μ can not be separated from the deviations of the CMM.

In case of the reduced three-rosette method for pitch calibrations, it can be shown that equation (22) is not only necessary, but also sufficient for the unique solvability, see [10, 11] for more details. In the present more complex situation of the three-rosette method in six DOF, a proof of a similar result is not known to the authors. However, the educated guess is that

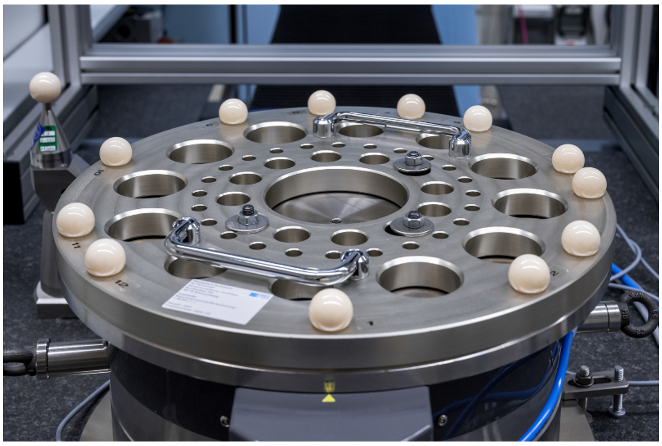

Figure 3. The new circular ball plate mounted on a rotary table. It can be seen that the spheres do not have uniform angular spacing.

condition (22) is necessary and sufficient for the solvability of equation (20) in case $R \geqslant 5$. If $R \leqslant 3$, the error separation is not possible for $N > R$, since there is not enough information to calculate all unknowns. If $R = 4$, in some situations condition (22) is fulfilled, but the unique solvability of equation (20) is still not given. In any case, for a given choice $\mathcal{R}$ of positions the unique solvability must be confirmed, e.g. by ensuring that the matrix on the left hand side of equation (20) is invertible.

After an appropriate choice $\mathcal{R}$ of positions is found, a ball plate with R balls at the angular positions $\frac{2\pi}{N} j_r$, $r = 1, \dots, R$, is necessary. Because of condition (22), in case that $R < N$ the balls can no longer be equally distributed on the plate. However, in general, it is not necessary to have two adjacent balls with the angular difference of $\frac{2\pi}{N}$; thus, even for very large N, the balls can be placed on the plate with sufficient space in between them.

At PTB, a ball plate with the following properties was designed and manufactured to measure rotary table deviations in steps of 5°:

- 12 balls are on a 5° angular grid, allowing the measurement of the rotary table deviations in steps of 5°. More precisely, the balls are at the angular positions 0°, 40°, 60°, 90°, 105°, 140°, 160°, 185°, 210°, 250°, 290° and 310°.
- 10 of the 12 balls are on a 10° angular grid, allowing rotary table deviations to be measured in steps of 10°.
- The positions of the 12 balls in the grid are optimized in such a way that the plate is suitable for measurements of 6, 8, or 12 balls for the 5° steps, and for the measurement of 8 or 10 balls for the 10° steps.

Optimisation should not be understood here in a strict mathematical sense. As mentioned before, the principal aim is to avoid ball positions which lead to especially large uncertainties.

Figure 3 shows the manufactured plate on a rotary table. The outer diameter of the plate is 400 mm, while the balls are located on a circle with a diameter of 370 mm. The precision spheres have a diameter of 25 mm and roundness deviations below 80 nm. The small form deviation of the spheres ensures

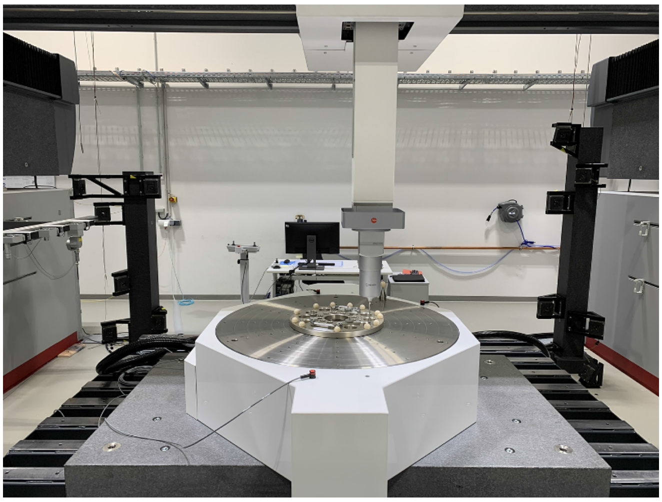

Figure 4. The new circular ball plate measured on the rotary table of a large CMM.

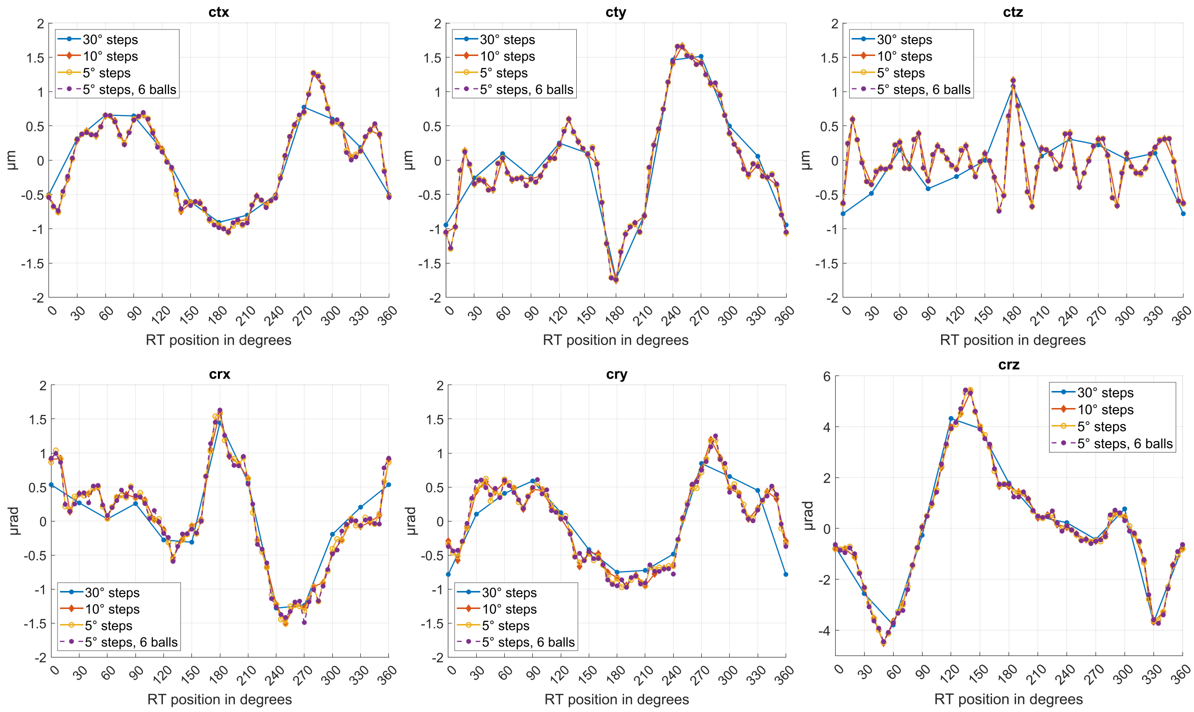

Figure 5. Rotary table deviations. The blue lines (solid line with dots) show results of measurements with a ball plate with 12 equidistant balls. The lines with the markers ○ show the results with the new ball plate, measured in 5° steps. The lines with the marker ◇ show the same measurement with the new ball plate, but evaluated in 10° steps. The dashed lines again show the same measurement with the new ball plate, but now only using the results of six balls for the evaluation.

that the calculated center is almost independent of the distribution of the probing points on the sphere.

4. Measurements

To verify the reduced method, measurements with the newly designed ball plate with 12 non-equidistant balls on the 5° angular grid were compared to measurements with the ball plate with 12 equidistant balls. The measurements were performed on a large CMM with a working volume of 5 m × 4 m × 2 m and a rotary table with a diameter of 1 m (figure 4). The balls were measured with 15 points distributed on the upper half of the sphere. It should be noted that about one year passed in between the measurements with the complete and reduced ball plates.

5. Results

The results of the measurements are shown in figure 5. The measurements in 5° steps were performed with the new ball plate with the 12 non-equidistant balls. This can be compared with the measurements in 30° steps at the corresponding angular positions. The results of the two measurements show good agreement. It can be further seen that the higher resolution significantly increases the information obtained for the rotary table. The measurement with the new ball plate was additionally evaluated using only the measurements of 6 of the 12 balls. The results show good agreement with the evaluation using the data from all 12 balls. Therefore, it may be sufficient to measure only 6 balls instead of 12. The same measurement of the new ball plate was also evaluated in 10° steps. Here, only the measurements of 10 balls and of 36 CMM positions were used. The difference to the measurements with the 5° steps is not significant for the rotary table investigated in this study. Therefore, it may also be an option to measure only the 10° steps for future measurements of this particular rotary table. However, the situation may be different for other rotary tables.

The expanded uncertainty ($k = 2$) for the single ball center measurements calculated via the residuals from the least squares fit yields 0.1 µm. This leads to uncertainties for the rotary table deviations calculated from the covariance matrix of about 0.03 µm for the translational deviations, about 0.23 µrad for the rotational deviations *crx* and *cry*, and 0.16 µrad for the positioning deviation *crz*. The actual uncertainties may be higher, since additional effects such as temperature induced drift are not included in the calculation. The differences between the measurements with the new and old ball plates are smaller than 0.1 µm for the linear errors, and smaller than 0.5 µrad for the rotational errors. This suggests that the uncertainties are about twice as large as the values given above.

6. Conclusions and outlook

The easy-to-use three-rosette method measures the deviations of rotary tables of CMMs directly at the location of use of the CMM. This method only requires an uncalibrated ball plate as additional equipment. Since it is an error separation method, the results of the three-rosette method are not affected by systematic errors of the linear axes of the CMM.

For the application of the reduced three-rosette method, a new ball plate was designed and manufactured to allow measurements of the rotary table errors in steps of 5°. When measuring all 12 balls on the plate, the measurement effort is six times lower than it would be using the complete three-rosette method (using a plate with 72 balls). It is possible to further reduce the measurement effort by (for example) measuring only six balls on the plate with only slightly increased measurement uncertainties.

The measurements performed with the new ball plate for the reduced three-rosette method on several CMMs show good agreement with previous full-method measurements taken with the old ball plate at common 30° angular grid points. Ongoing work undertaken together with a manufacturer of rotary tables concerns comparison with alternative measurement methods. As for the complete method described in [7], good agreement has already been achieved for the position error *crz*.

Future work will also focus on the application of the method to machine tools.

Data availability statement

The data that support the findings of this study are openly available at the following URL: https://doi.org/10.5281/zenodo.7642531.

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