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A Substitution Method for Partial Differential Equations Using Ramadan Group Integral Transform

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

In this paper we introduce the concept of Ramadan Group integral transform substitution (RGTS) method to solve some types of Partial differential equations. This new method is a convenient way to find exact solution with less computational cost as compared with method of separation of variables (MSV) and variation iteration method (VIM). The proposed method solves linear partial differential equations involving mixed partial derivatives.

Keywords: Nonlinear partial differential equations; Ramadan Group transformation; Adomian decomposition method; Laplace substitution method.

1 Introduction

Nonlinear partial differential equations (NLPDEs) involving mixed partial derivatives are mathematical models that are used to describe complex Phenomena arising in the world around us. The nonlinear equations

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appear in many applications of science and engineering such as fluid dynamics, plasma physics, hydrodynamics, solid state physics, optical fibers and other disciplines. In the recent years, many authors mainly had paid attention to study solutions of NLPDEs by using various methods, for example variation iteration method, Adomian decomposition [1] and Laplace substitution method [2]. Partial differential equations have big importance in Mathematics and other fields of science. Therefore, it is very important to know methods to solve such partial differential equations. One of the most known methods to solve partial differential equations is the Ramadan Group transform (RGT) method [3,4] which is considered to be the generalization of the known integral transforms as Laplace transform method [5,6,7,8] and Sumudu transform method [9]. The key motivation for pursuing theories for integral transforms is that it gives a simple tool which is represented by an algebraic problem in the process of solving differential equations. An intrinsic structure and properties of Laplace-typed integral transforms , see Hwajoon Kim [10].

In this paper we try to solve some types of partial differential equations using the proposed RGTS method. In the following section we give a summary for RGT.

2 Ramadan Group Transformation (RGT) [3,4]

A new integral Ramadan Group transform (RGT) defined for functions of exponential order, was proclaimed. We consider functions in the set A, defined by:

$$A = \{ f(t) : \exists M, t_1, t_2 \succ 0 s.t. \left| f(t) \right| \prec M e^{\frac{|t|}{t_n}}, if t \in (-1)^n \times [0, \infty) \}$$

The RG transform is defined by

$$K(s,u) = RG[f(t);(s,u)] = \begin{cases} \int_0^\infty e^{-st} f(ut)dt & -t_1 \prec u \leq 0 \\ \int_0^\infty e^{-st} f(ut)dt & 0 \leq u \prec t_2 \end{cases}$$

This transform which is a generalization of Laplace and Sumudu transforms is introduced by M.A. Ramadan et al. [3,4] and, accidentally and unpredictably, it was also introduced by Z. H. Khan and W. A. Khan [11] under the name of N-Transform. A theoretical study of this natural transform is investigated by F. Belgacem and R. Silambarasan [12]. Further investigations for the new integral transform on time scales and its applications is considered by H. A. Agwa et al. [13].

Consider $F(s) = L(f(t) = \int_0^\infty e^{-st} f(t)dt$ and $G(u) = \int_0^\infty e^{-t} f(ut)dt$, are the Laplace and Sumudu integral transforms respectively, then we can write the following theorem.

Theorem 1 [3]

$$K(s,1) = F(s) \quad , \; K(1,u) = G(u) \, , \quad K(s,u) = \frac{1}{u} F(\frac{s}{u})$$

Theorem 2 [3]

suppose K(s,u) is the Ramadan Group transform of the function $f(t)$ then

$$RG(\frac{df(t)}{dt}) = \frac{sRG(f(t) - f(0)}{u} \qquad , \qquad RG(\frac{d^2 f(t)}{dt^2}) = \frac{s^2 RG(f(t)) - sf(0) - uf'(t)}{u^2}$$

And in general $RG(\frac{df^n}{dt^n}) = \frac{s^n RG(f(t))}{u^n} - \sum_{k=0}^{n-1} \frac{s^{n-k-1} f^{(k)}(0)}{u^{n-k}}$

Ramadan Group transform of some functions

$F(t)$	$RG(f(t)) = K(s,u)$
1	$K(s,u) = \frac{1}{s}$
t	$K(s,u) = \frac{u}{s^2}$
$\frac{t^{n-1}}{(n-1)!}$	$K(s,u) = \frac{u^{n-1}}{s^n}$
$\frac{1}{\sqrt{\pi t}}$	$K(s,u) = \frac{1}{\sqrt{su}}$
e^{at}	$K(s,u) = \frac{1}{s - au}$
te^{at}	$k(s,u) = \frac{u}{(s - au)^2}$
$\frac{\sin(wt)}{w}$	$K(s,u) = \frac{u}{s^2 + u^2 w^2}$
$\cos(wt)$	$K(s,u) = \frac{s}{s^2 + u^2 w^2}$
$\frac{\sinh(at)}{a}$	$K(s,u) = \frac{u}{s^2 + u^2 a^2}$

The main goal of this paper is to describe new method for solving linear partial differential equations involving mixed partial derivatives. This powerful method will be proposed in Section 3; in Section 4 we will apply it to some examples and in last section we give some conclusion.

3 Ramadan Group Integral Transform Substitution (RGTS) Method

The aim of this section is to discuss the Ramadan Group integral transform substitution method. We consider the general form of non homogeneous partial differential equation with initial conditions is given below

$$Lu(x, y) + Ru(x, y) = h(x, y) \tag{3.1}$$

$$u(x,0) = f(x). \qquad u_y(0, y) = g(x). \tag{3.2}$$

Here $L = \frac{\partial^2}{\partial x \partial y}$, $Ru(x, y)$ is the remaining linear term in which contains only of first order partial derivatives of $u(x, y)$ with respect to either $x \, or \, y$ and $h(x, y)$ is the source term. We can write equation (3.1) in following form

$$\frac{\partial^2 u}{\partial x \partial y} + Ru(x, y) = h(x, y)$$

$$\frac{\partial}{\partial x}(\frac{\partial u}{\partial y}) + Ru(x, y) = h(x, y) \quad (3.3)$$

Putting $\frac{\partial u}{\partial y} = U(x, y)$ in equation (3.3), we get

$$\frac{\partial U}{\partial x} + Ru(x, y) = h(x, y) \quad (3.4)$$

Taking Ramadan Group transform of equation (3.4) with respect to x, we get

$$\frac{sRG_x[U(x, y)] - U(0, y)}{u} = RG_x[h(x, y) - Ru(x, y)]$$

$$RG_x[U(x, y)] = \frac{1}{s}U(0, y) + \frac{u}{s}RG_x[h(x, y) - Ru(x, y)]$$

$$RG_x[U(x, y)] = \frac{1}{s}u_y(0, y) + \frac{u}{s}RG_x[h(x, y) - Ru(x, y)]$$

$$RG_x[U(x, y)] = \frac{1}{s}g(y) + \frac{u}{s}RG_x[h(x, y) - Ru(x, y)] \quad (3.5)$$

Taking inverse Ramadan Group transform of equation (3.5) with respect to x, we get

$$U(x, y) = g(y) + RG_x^{-1}[\frac{u}{s}RG_x[h(x, y) - Ru(x, y)]] \quad (3.6)$$

Resubstitute the value of U(x, y) in equation (2.6), we get

$$\frac{\partial u(x, y)}{\partial y} = g(y) + RG_x^{-1}[\frac{u}{s}RG_x[h(x, y) - Ru(x, y)]] \quad (3.7)$$

This is a first order partial differential equation in the variables x and y.

Taking Ramadan Group transform of equation (3.7) with respect to y, we get

$$sRG_y[u(x, y)] = f(x) + uRG_y[g(y) + RG_x^{-1}[\frac{u}{s}RG_x[h(x, y) - Ru(x, y)]]$$

$$RG_y[u(x, y)] = \frac{1}{s}f(x) + \frac{u}{s}RG_y[g(y) + RG_x^{-1}[\frac{u}{s}RG_x[h(x, y) - Ru(x, y)]] \quad (3.8)$$

Taking the inverse Ramadan Group transform of equation (3.8) with respect to y, we get

$$u(x, y) = f(x) + RG_y^{-1}[\frac{u}{s}RG_y[g(y) + RG_x^{-1}[\frac{u}{s}RG_x[h(x, y) - Ru(x, y)]]] \quad (3.9)$$

The last equation (3.9) gives the exact solution of initial value problem (3.1).

In the following section we will apply RGT for Partial Differential Equations.

4 Applications of RGTS Method

Example 1.

Consider the partial differential equation

$$\frac{\partial^2 u}{\partial x \partial y} = e^{-y} \cos x \tag{4.1}$$

With the initial conditions

$$u_y(0, y) = 0, \quad u(x,0) = 0 \ , \tag{4.2}$$

and general linear term $h(x, y) = e^{-y} \cos x$, $Lu(x, y) = \frac{\partial^2 u}{\partial x \partial y}$, Equation (4.1) we can write in the following form

$$\frac{\partial}{\partial x}(\frac{\partial u}{\partial y}) = e^{-y} \cos x, \tag{4.3}$$

In Equation (3.3), substantiation $\frac{\partial u}{\partial y} = U$, we get

$$\frac{\partial U}{\partial x} = e^{-y} \cos x, \tag{4.4}$$

This is the non homogeneous partial differential equation of first order. Taking Ramadan Group transform on both sides of equation (4.4) with respect to x, we get

$$\frac{sRG_x[U(x, y)] - U(0, y)}{u} = RG_x[e^{-y} \cos x]$$

$$RG_x[U(x, y)] = e^{-y} \frac{u}{s}(\frac{s}{s^2 + u^2}) \tag{4.5}$$

Taking inverse Ramadan Group transform of equation (4.5) with respect to x, we get

$$U(x, y) = e^{-y} \sin x \quad . \quad \text{That is , } \frac{\partial u(x , y)}{\partial y} = e^{-y} \sin x \tag{4.6}$$

This is the partial differential equation of first order in the variables x and y. Taking Ramadan Group integral transform of equation (4.6) with respect to y, we get

$$\frac{sRG_y[u(x, y)] - u(x,0)}{u} = RG_y[e^{-y} \sin x]$$

$$RG_y[u(x, y)] = \frac{u}{s(s+u)} \sin x \tag{4.7}$$

Taking inverse Ramadan Group integral transform of equation (4.7) with respect to y, we get

$$u(x,y) = \sin x(1 - e^{-y}) \tag{4.8}$$

This is the required exact solution of equation (1). This can be verifying though the substitution.

Example 2. Consider the partial differential equation

$$\frac{\partial^2 u}{\partial x \partial y} = \sin x \sin y \tag{4.9}$$

with the initial conditions

$$u_y(0,y) = -2\sin y \;\; and \;\; u(x,0) = 1 + \cos x \tag{4.10}$$

In the above example assume that both $u_y(x,y)$ *and* $u_x(x,y)$ are differentiable in the domain of definition of function u (x ,y) [Young's Theorem]. This implies that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

Given initial conditions (4.10) force to write the equation (4.9) in the following form and use the substitution.

$$\frac{\partial u}{\partial y} = U \tag{4.11}$$

$$\frac{\partial U}{\partial x} = \sin x \sin y \tag{4.12}$$

This is the non homogeneous partial differential equation of first order. Taking Ramadan Group transform on both sides of equation (4.11) with respect to x, we get

$$\frac{sRG_x[U(x,y)] - U(0,y)}{u} = RG_x[\sin x \sin y]$$

$$RG_x[U(x,y)] = \frac{-2}{s}\sin y + \frac{u}{s}\sin y[\frac{u}{s^2+u^2}]$$

$$RG_x[U(x,y)] = -\sin y(\frac{2}{s} - [\frac{u^2}{s(s^2+u^2)}]) \tag{4.13}$$

Taking inverse Ramadan Group transform of equation (4.13) with respect to x, we get

$$U(x,y) = -\sin y(1+\cos x)$$

$$\frac{\partial u(x,y)}{\partial y} = -\sin y(1+\cos x) \tag{4.14}$$

This is the partial differential equation of first order in the variables x and y. Taking Ramadan Group transform of equation (4.14) with respect to y, we get

$$\frac{sRG_y[u(x,y)]-u(x,0)}{u}=-\frac{u}{s^2+u^2}(1+\cos x)$$

$$RG_y[u(x,y)]=\frac{s}{(s^2+u^2)}(1+\cos x) \quad (4.15)$$

Taking inverse Ramadan Group integral transform of equation (4.15) with respect to y, we get

$$u(x,y)=\cos y(1+\cos x)$$

This is the required exact solution of equation (4.9). This can be verifying though the substitution.

Example 3. Consider the partial differential equation

Consider the partial differential equation

$$\frac{\partial^2 f}{\partial x \partial y}+e^{x-y}=0 \quad (4.16)$$

with initial conditions

$$f_y(0,y)=-e^{-y} \quad , \quad f(x,0)=e^x \quad (4.17)$$

Equation (4.16) we can write in the following form

$$\frac{\partial}{\partial x}(\frac{\partial f}{\partial y})=-e^{x-y} \quad (4.18)$$

Putting $\frac{\partial f}{\partial y}=U$ in equation (4.18), we get

$$\frac{\partial U}{\partial x}=-e^{x-y} \quad (4.19)$$

This is the non homogeneous partial equation of first order. Taking Ramadan Group transform on both sides of equation (4.19) with respect to x, we get

$$\frac{sRG_x[U(x,y)]-U(0,y)}{u}=-\frac{e^{-y}}{s-u}$$

$$RG_x[U(x,y)]=\frac{-e^{-y}}{s}-\frac{u}{s(s-u)}e^{-y}$$

$$RG_x[U(x,y)]=-\frac{e^{-y}}{s-u} \quad (4.20)$$

Taking inverse Ramadan Group transform of equation (4.20) with respect to x, we get

$$U(x,y) = -e^{x-y}$$

$$\frac{\partial f(x,y)}{\partial y} = -e^{x-y} \tag{4.21}$$

Taking Ramadan Group transform of equation (4.21) with respect to y, we get

$$\frac{sRG_y[f(x,y)] - f(x,0)}{u} = -\frac{e^x}{s+u}$$

$$RG_y[f(x,y)] = \frac{e^x}{s} - \frac{u}{s(s+u)}e^x \tag{4.22}$$

Taking inverse Ramadan Group transform of equation (4.22) with respect to y, we get

$$f(x,y) = e^{x-y} \tag{4.23}$$

This is the require d exact solution of equation (4.16) which can be verified through the substitution.

Example 4. Consider the partial differential equation

$$\frac{\partial^2 f}{\partial x \partial y} = e^x + e^x \cos y \tag{4.24}$$

With initial conditions

$$f_y(0,y) = 1 \quad , \qquad f(x,0) = 0 \tag{4.25}$$

Equation (4.24) we can write in the following form

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = e^x + e^x \cos y \tag{4.26}$$

Putting $\frac{\partial f}{\partial y} = U$ in equation (4.26), we get

$$\frac{\partial U}{\partial x} = e^x + e^x \cos y \tag{4.27}$$

This is the non homogeneous partial equation of first order. Taking Ramadan Group transform on both sides of equation (4.27) with respect to x, we get

$$\frac{sRG_x[U(x,y)] - U(0,y)}{u} = \frac{1}{s-u} + \frac{\cos y}{s-u}$$

$$RG_x[U(x,y)] = \frac{1}{s} + \frac{u}{s(s-u)} + \frac{u}{s(s-u)}\cos y \quad (4.28)$$

Taking inverse Ramadan Group transform of equation (4.28) with respect to x, we get

$$U(x,y) = e^x + (e^x - 1)\cos y$$

$$\frac{\partial f}{\partial y} = e^x + (e^x - 1)\cos y \quad (4.29)$$

Taking Ramadan Group transform of equation (4.29) with respect to y, we get

$$\frac{sRG_y[f(x,y)] - f(x,0)}{u} = \frac{e^x}{s} + \frac{s}{s^2+u^2}(e^x - 1)$$

$$RG_y[f(x,y)] = \frac{u}{s^2}e^x + \frac{u}{s^2+u^2}(e^x - 1) \quad (4.30)$$

Taking inverse Ramadan Group transform of equation (4.30) with respect to y, we get

$$f(x,y) = ye^x + \sin y(e^x - 1) \quad (4.31)$$

This is the required exact solution of equation (4.24). Which can be verify through the substitution.

5 Conclusion

In this paper, Ramadan Group Transform Substitution (RGTS) Method is applied to solve partial differential equations in which involves the mixed partial derivatives and general linear term. The proposed method is a generalization of any integral transform substitution available methods for this type of considered problems.

Competing Interests

Authors have declared that no competing interests exist.

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